DOCUMENT RESUME

ED 121 452 PS 008 510

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TITLE Developmental Mathematics and the Young Child: A Piaget Rationale.
PUB DATE 75
NOTE 12p.

EDRS PRICE MF-$0.83 HC-$1.67 Plus Postage
DESCRIPTORS Arithmetic; Classification; *Early Childhood Education; *Intellectual Development; *Mathematics Curriculum; *Mathematics Education; *Mathematics Instruction; Number Concepts; Object Manipulation; Serial Ordering; Spatial Relationship; Teaching Techniques; Thought Processes; Topology
IDENTIFIERS Developmental Stages; *Piaget (Jean)

ABSTRACT

This article discusses the challenge mathematics educators face in deciding what and how mathematics is to be taught, and offers some suggestions for teachers of young children based on Piaget's developmental theory of mathematics. Piaget's cognitive stages are briefly described and the concrete stage, which spans the age range included in early childhood education, is singled out for detailed examination. Within this stage, three areas of knowledge are dealt with: (1) physical, (2) spatial-temporal, and (3) logico-mathematical. Physical knowledge is described as deriving from the observations of physical phenomena in the environment. The primary recommendation for fostering this type of knowledge in the classroom is to provide plenty of opportunity for object manipulation. Spatial-temporal knowledge is described as thinking structures reflecting concepts of space and time and topological activities are suggested to foster its development in the classroom. Logico-mathematical knowledge is described as dealing with the study of relationships between and among objects. Activities involving classification, seriation, number concepts, and arithmetic operations are recommended at the classroom level. Finally, four benefits of a developmental view of mathematics are noted. (JMB)

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DEVELOPMENTAL MATHEMATICS AND THE YOUNG CHILD: A PIAGET RATIONALE

by

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Developmental Mathematics and the Young Child: A Piaget Rationale

Resonating throughout schools and classrooms in many school systems--both large and small, the cry of "Back to Basics" is being heard more and more frequently and with greater volume than ever before. "Return to the three R's", "Skill Mastery", "Good ol' 'rithmetic, reading, and 'riting" has prompted many school districts across the land to reassess their school curriculums and to evaluate the students' achievements in learning the basic skills on district- and state-wide bases

The feeling for the need to "return to the basics" is not singular to secondary, middle schools, or intermediate grade classrooms. Also, at the early childhood level, preschool, kindergarten, through third grade, greater emphasis is also being shown for skill learning in mathematics, reading, and writing. Academic learning is <u>in</u> and learning through play is <u>out</u>!

Certainly a return to emphasis on mastery of basic skills is not necessarily incompatible with retaining and improving upon contemporary and innovative approaches in curriculum and instruction. The burden of proof, however, is now on educators at all levels to prove it to school and community members and leaders. In particular, with the passing of the "New Math" era, that was heralded in so strongly just a few years ago, how are educators regrouping and meeting this challenge? Specifically, how will early educators charged with the learning of young children in preschools, kindergarten, and the primary grades act and react in an effort to meet this challenge? One immediate reaction is to throw the "baby out with the bath water"--return to traditional memorization and stress recall of mathematical facts to the exclusion of all "New Math" principles. A more thought-

ful approach, however, is to examine existing alternatives with the focus upon retaining and using contemporary principles of mathematics education in conjunction with the back to basics philosophy. Essentially, the outcome could be the "best of two current worlds" with a continuing emphasis to develop theory and application for better mathematics programs. One of the many options for early educators as well as those at all levels, yet within the "best of two worlds" alternative is to examine what is currently known about contemporary research in mathematics instruction and in human growth and development. In the context of mathematics, school curriculum, and early childhood classrooms, another way of stating the option is, "How can young children learn and acquire mathematic principles and basic mathematical skills?" Among the number of choices that are also available to early educators on "how young children learn" is one that is recently receiving a great amount of attention from both researchers and practitioners. This option, developed by Professor Jean Piaget at the University of Geneva-Switzerland, is being tested and applied in early childhood classrooms across the nation.

Piaget's Developmental Theory of Mathematics

Piaget has developed a theory of growth and learning in human organisms from birth through adulthood and onward covering the entire life span. The emphasis or focus of this theory is upon evolving thought structures or the thinking processes. Regarding the active involvement of a child within an environment as crucial to and a facilitator of the thinking processes, this interaction is the source for the organization-reorganization, and subsequent internalization of these thought or mental modes of action.

Piaget, viewing intellectual development in relation to the child's adaptive actions within an environment, sees assimilation and accommodation as the twin complimentary processes that comprise the adaptive actions and provide the means for the organization, reorganization, and internalization of the thought structures. Assimilation is a construct that refers to physical and psychological processes through which the child incorporates environmental objects into the mind (Yawkey, 1975). Accommodation, also a construct, denotes physical and psychological processes in which the thinking structures of the child become modified to meet the requirements of the environment that has already been assimilated (Yawkey, 1975). "Intellectual adaptation is thus a process of achieving a state of balance between the assimilation of an experience into deductive structures and the accommodation of those structures to the data of experience." (Piaget, 1971, p. 14). In no way does Piaget deny the existence of social and emotional growth processes as some critics have charged. Social and emotional processes, since they both contain large components that are cognitive in origin, are viewed through the contributions they make to constructing and facilitating development of the thought processes.

For Piaget, the growth of thinking structures develop and proceed in heirarchical fashion from simple to complex and from concrete to abstract through four structurally distinct stages. Consistent with Piaget's focus, each one of the cognitive stages show qualitatively different thought structures. Together with suggested ages, the stages as analyzed by Hunt (1971) are:

1. Sensorimotor Stage of Intellectual Development (birth through 18 months to 2 years)

2. Concrete Stage of Intellectual Development (2 through 11/12 years of age)
3. Formal Operations Stage of Intellectual Development (11/12 through 15/16 years of age and onward)

The age ranges per stage are relative and not absolute and depend upon the factors of the child's background experiences with the environment, broad, yet undefined, maturational aspects, cultural and societal patterns transmitted to the child by family and school. The rate at which children develop and acquire the thought structures varies, but the sequences of development within and between the stages is constant or invariant across individuals regardless of society and culture.

Early childhood education, preschool through third grade, clearly falls within the Concrete Stage of Intellectual Development and the focus on this as in the previous and later stage is the evolving and developing thought structures. Given the emphasis on thinking processes, placed in the context of mathematics education, the early educator to further examine this option as a viable approach to meeting current challenge can logically ask, "How does the developmental notion of thinking structures relate to mathematics curriculum and instruction of young children in the Concrete Stage of Intellectual Growth? Specifically, Piaget suggests that we examine different modes of structuring knowledge based not upon traditionally recognized content and subject disciplines but upon areas of knowledge characteristic of children's adaptive actions necessary for developing thinking processes. Among those areas of knowledge reflective of thinking structures in the Concrete Stage, the young child and of mathematics curriculum, are those that Piaget has identified as: (1) physical knowledge; (2) spatio-temporal knowledge; and

(3) logico-mathematical knowledge.

Physical knowledge, structured from thought processes, develop most effectively through the feedback the child receives from objects in the external environment. Developmentally speaking, observed attributes of physical objects and phenomena are fundamental and crucial to physical knowledge. In a mathematical sense, the key attributes of objects important in building thought structures, include color, shapes, texture, cardinal number, dimensions of width and length, and many others. Physical knowledge also denotes observations of physical phenomena in the environment and the reactions of them gained from object manipulation. In a developmental and mathematical sense, the emphasis here concerns the child's response repertoires used in investigating and exploring objects as well as the mathematics content derived from this exploration and manipulation. Children can come to better know the mathematical attributes of objects only if they have the opportunities to make a number and variety of manipulations on the objects within the environment. The greater variety and number of responses the child makes to any object, the better he or she will come to know the attributes of that object, and the larger response repetoire that will develop. Mathematically speaking, the child will not only better organize, reorganize, and internalize the thought processes for attributes explored, but also from a developmental point of view, begin to categorize observed reactions of the objects manipulated. At a classroom level of operation, a teacher will need to provide much opportunity for the children to employ as many and varied actions on objects and to observe reactions of physical phenomena in order to thoroughly and

soundly learn selected mathematical attributes. As a child acts upon, observes, and notes the reactions of objects, he or she will better able to identify, describe, and classify the mathematic-physical properties of them as well as other objects that are novel and unfamiliar. In other words, the child will not only be better able to acquire but also to generalize thinking structures of "attribute-knowing" from one situation to another.

Spatio-temporal thinking is a more advanced level of thought development than those categorized by physical knowledge. From a mathematical view, this knowledge area comprises thinking structures reflecting concepts of space and time. Developmentally speaking, thinking structures reflective of space and time are acquired, like physical knowledge, from feedback from external reality, and are based upon manipulation and observation of stimulus events and reactions of physical phenomena. However, this area of knowledge also includes space and time relationships between physical objects and phenomena. Based upon thinking structures evolving from feedback from the environment coupled with Piaget's notion of acquiring relationships of space and time, mathematical experiences of closed and open curves or paths and other related aspects of "rubber" or topological geometry are fundamental. At the classroom level, mathematical experiences with open and closed curves in the context of topological relationships of enclosure or surrounding including the concept of "betweeness", proximity or nearness of an object to other objects, separation or distinguishing one object from another or parts of objects from others, and order of objects--linear and nonlinear--are to be provided. These experience-oriented activities also include arranging

and rearranging objects to fit within other objects and concrete experiences of predicting paths objects will take when thrown. Knowledge of space through concrete experiences with topological geometry, for Piaget, are prerequisites to successful acquisition of Euclidean geometry. Thinking structures, related to time, a part of spatio-temporal knowledge, deal with learning about a unit of time, time sequences or order of units and then intervals of time.

In the concrete stage of intellectual development, the last area of knowledge related to mathematics education is logico-mathematical. From a developmental point of view, logico-mathematical thought structures are developed and acquired through further exploration within environments, but the child does not receive feedback from objects in a physical sense. Here, knowledge gained, is acquired through coordination of actions and the results of these coordinated actions. "The properties of objects exist in objects, but the relationship among them does not." (Kamii, 1972, p. 155). On a mathematical plan of discourse, the coordination of actions and their results refer to the study of relationships between and among objects. At the classroom level, after identifying and describing attributes, examining space and time relationships, the child now has the mathematical background to further acquire and establish relationships between and among objects. Thus, classification-grouping objects according to similar as well as non-similar characteristics, ordination, or "which one", seriation or ordering objects by various attributes, as well as cardinal number are developed. Other mathematical experiences relative to building logico-mathematical

thought structures include correspondence, equivalence, equalities, the set relations of "same number as", "greater" and "fewer number than", union and intersection of sets, addition (and its inverse-subtraction), and multiplication (and its inverse operation-division). Although the order of teaching for logico-mathematical thought structures will inevitably vary within this knowledge area from school to school, these preliminary relationships between and among objects are prerequisites to other advanced arithmatecal learning of the intermediate, middle, and high school mathematics.

In Retrospect . . .

Facing the mathematics educator with passing of the "New Math" era are a number of challenges and decisions that will be made about "what" and "how" mathematics is to be taught. With this current push "back-to-basics", and other challenges like them, alternatives and options to mathematics curriculum and instruction must be carefully and thoroughly examined.

At the early childhood, as well as other levels of the school, the option of viewing mathematics in context of developing thought processes, characteristic of Piaget's theory, and teaching for these mathematical thinking structures can be explored as one of the ways to regroup and successfully meet these contemporary challenges. The benefits of a developmental view of mathematics are many. Chief among them are:

1. The teaching of mathematics and much of its content can be supported in the context of human growth and development through the acquisition of thinking structures.
2. Mathematics in the context of Piaget's developmental theory is placed on a firm research base which supports the need for manipulation, use of concrete objects, suggests that feedback to the child is a requirement for learning, and stresses a partial heirarchy or teaching order of mathematics' concepts.
3. Important mathematics principles are not taught for the sake of facts, but for organizing, reorganizing, and systematizing mathematical thinking structures.
4. Significant principles of curriculum and instruction in mathematics derived from the "New Math" era can easily and flexibly be coupled with a Piagetian orientation and then modified to fit contemporary requirements of the school and community.

Although Piaget's theory of intellectual development requires more research support and experimentation, viewing mathematics development in the context these learning principles can result in better mathematics programs and sound acquisition of arithmatical concepts and skills.

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